UK Junior Mathematical Olympiad 2004 Solutions

A1 **1.75** $1 + \dfrac{1}{1 + \frac{1}{3}} = 1 + \dfrac{1}{\frac{4}{3}} = 1 + \frac{3}{4} = 1.75.$

A2 **Zara** The first sentence tells us that Gus is not the youngest. The second sentence tells us that neither Alessandro nor Flora is the youngest. From the third sentence, Oliver is not the youngest, whilst the sixth sentence tells us that Yvette is not the youngest. So Zara must be the youngest and the order of their ages is Gus, Yvette, Flora, Alessandro, Oliver, Zara.

A3 **3736** On Tuesday, the Pied Piper caught 900 rats.
On Wednesday he caught 1080 rats ($900 + 180$).
On Thursday he caught 756 rats ($1080 - 324$).
So in total, the number of rats caught is $1000 + 900 + 1080 + 756 = 3736$.

A4 **£240** The difference between one third of the sum of money and one quarter of it is £20. So one twelfth of the sum of money is £20.

A5 **81s** Note that there are 9 gaps between the lamps, so the total time is $(5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13)$ seconds = 81 seconds.

A6 **45** It next tells the correct time after gaining 12 hours, i.e. 720 minutes. So the number of days is $720 \div 16$.

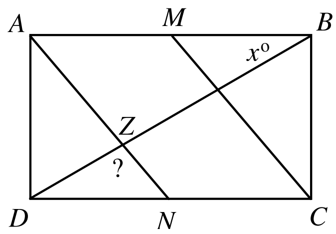
A7 **−3** Let the number be x. Then $x^3 = 9x$, so either $x = 0$ or $x^2 = 9$, giving $x = 3$ or $x = -3$. Of these three values of x, only -3 is also a solution of the equation $x^2 = x + 12$.

A8 **3 : 5** When 2 litres of oil are added to the pan, the volume of oil increases from one half of the volume of water to two thirds of the volume of water, i.e. by one sixth of the volume of water. So at that stage there are 12 litres of water in the pan and, therefore, 8 litres of oil. Hence originally the pan contained 10 litres of water and 6 litres of oil. So the original ratio of oil to water was $6 : 10 = 3 : 5$.

A9 **$4/\pi$** The circumference of the circle is 2π. This is the distance its centre moves each time the circle rolls for one revolution. When the circle rolls from one corner to an adjacent corner, its centre moves a distance 2, so the circle makes $1/\pi$ revolutions. As it needs to do this four times before the circle returns to its original position, the number of revolutions is $4/\pi$.

A10 **7** The four youngest must receive at least $1 + 2 + 3 + 4 = 10$ sweets, so the oldest receives at most 10 sweets. Also, as $5 + 4 + 3 + 2 + 1 = 15$, which is less then 20, the oldest must receive at least 6 sweets. Systematically listing the different ways of sharing the sweets gives (10, 4, 3, 2, 1), (9, 5, 3, 2, 1), (8, 6, 3, 2, 1), (8, 5, 4, 2, 1), (7, 6, 4, 2, 1), (7, 5, 4, 3, 1) and (6, 5, 4, 3, 2).

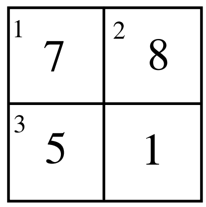
B1 As N is the midpoint of DC, DN has length 1; so triangle ADN is an isosceles right-angled triangle with $\angle DNA = \angle DAN = 45°$. As AB is parallel to DC, $\angle BDC = \angle DBA = x°$ (alternate angles). So, in triangle DZN, $\angle DZN = (180 - 45 - x)° = (135 - x)°$.



B2 Let the lengths of a short and a long side of a card be x and y respectively. Then rectangle A measures x by $3y$ and has perimeter $2x + 6y$; rectangle B measures $3x$ by y and has perimeter $6x + 2y$.

So $2x + 6y = 2(6x + 2y)$ i.e. $2x + 6y = 12x + 4y$ i.e. $10x = 2y$ giving $x : y = 1 : 5$.

B3 As 1 Down is a multiple of 25, it is 25, 50 or 75. However, it cannot be 50 as none of the numbers can begin with 0, so 3 Across starts with 5 and is therefore 51 (3×17) or 57 (3×19). As 2 Down is a square, its unit digit cannot be 7, so 3 Across must be 51. The only two-digit square ending in 1 is 81, so 2 Down is 81. We now have 28 and 78 as possible solutions to 1 Across, but of these only 78 is a multiple of 3. So 1 Across is 78 and 1 Down is 75.

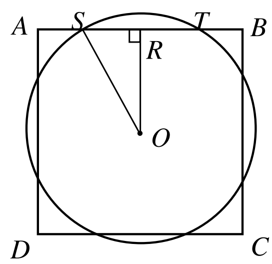


B4 Let the centre of the circle, and hence also of the square, be O and let the midpoint of ST be R.

Let the square have side $4x$. As OR is perpendicular to AB, ORS is a right-angled triangle in which $OR = 2x$ and $RS = x$. Therefore $OS^2 = OR^2 + RS^2 = 4x^2 + x^2 = 5x^2$.

So the radius of the circle is $\sqrt{5}x$ and its area is $\pi \times 5x^2 = 5\pi x^2$.

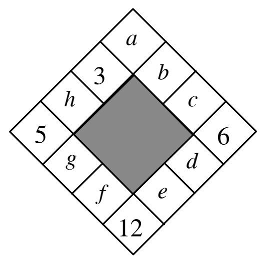
Now the area of the square $ABCD$ is $(4x)^2 = 16x^2$. As $16 > 5\pi$ (≈ 15.7), square $ABCD$ has the larger area.



B5 Child A scores 24 points, so wins four events and comes second in one other, which must be event W, won by E. The total number of points awarded is 75, so B, C, D and E score 51 points between them. Child E has a total of 8 points from 2 events, so must score at least 11 points overall. Hence B, C and D have minimum total scores of 14, 13 and 12 respectively. Now $14 + 13 + 12 + 11 = 50$, so we can deduce that the totals are B:15; C:13; D:12; E:11. Child C does not win any events, so could not obtain 13 points by scoring 1 point or 2 points in each of four events, so C wins 1 point in one event and 3 points in each of the others. These are V, W, Y and Z, as we know that E scores 3 points in event X. Child E has 8 points from two events and a total of 11 points, so scores 1 point in V, Y and Z. Child D has 4 points from V and a total of 12 points, so scores 2 points in each of W, X, Y and Z. We now know the points scored by each competitor, with the exception of child B, in each event so we may now conclude that B scored 2, 1, 4, 4 and 4 points in events V, W, X, Y and Z respectively, giving a total of 15 as stated earlier.

	V	W	X	Y	Z	Total
A	5	4	5	5	5	24
B	2	1	4	4	4	15
C	3	3	1	3	3	13
D	4	2	2	2	2	12
E	1	5	3	1	1	11

B6 Let the numbers to be placed in the squares be a, b, c, d, e, f, g and h, as shown. The sum of the whole numbers from 1 to 12 inclusive is 78, so the total of the numbers along the four edges is $78 + 5 + 12 + 6 + a$, since each of the four corner squares 'belongs' to two edges. So $101 + a$ must be divisible by 4 if each edge is to have the same total, which means that $a = 3$, 7 or 11. However, 3 has been used already, so $a = 7$ or 11.



If $a = 7$, the four edges total 108, so each edge totals 27. This would require h to be 12, but 12 has been used already. Therefore a cannot be 7.

If $a = 11$, the four edges total 112, so each edge totals 28. This requires h to be 9.

We now have $b + c = 28 - 17 = 11$; $d + e = 28 - 18 = 10$; $f + g = 28 - 17 = 11$ and the six numbers left are 1, 2, 4, 7, 8 and 10. Of these, only 2 and 8 total 10, so $d = 8$ and $e = 2$ or vice-versa. We now have two pairs of numbers (1, 10) and (4, 7) which total 11. So b could be any one of these four numbers. Once the value of b is fixed, so is the value of c, but there are still two possibilities for f and g. For instance, if $b = 1$, then $c = 10$; this leaves $f = 4$ and $g = 7$ or vice-versa.

So for each of the four possible values of b, the values of c, f and g can be chosen in two different ways, giving eight ways of choosing b, c, f and g. For each of these, the values of d and e can be chosen in two ways, so the total number of possibilities is 16.